Seat No. : _______________

ZO-129

May-2014

M.Sc. Sem.-II

STA : 408 (Distribution Theory)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions carry equal marks.
(2) Scientific calculator can be used.

1. (a) Let $x_1, x_2, \ldots x_n$ are independent discrete random variables and N is also a random variable independent of x_i's. Let $Y = \sum_{i=i}^{n} X_i$ and $\Phi_i = 1, 2, 3$ are the characteristic functions of random variables N, X and Y respectively. Express characteristic function Φ_3 as a compound function of Φ_1 and Φ_2.

OR

Define Neyman type – A distribution. Obtain its probability generating function. Hence derive its r^{th} factorial cumulant. Also describe the method of fitting of Neyman type – A distribution to the numerical data.

(b) Define Poisson – Binomial distribution. Obtain its probability generating function. Show that Poisson – Binomial distribution tends to Poisson – Poisson distribution. State necessary assumptions involved.

OR

Define Poisson – Pascal distribution. Obtain recurrence relations for probabilities and descending factorial moments for this distribution.

2. (a) Discuss the roll of non-central distributions in statistical inference with illustration. If $X \sim N(\mu, 1)$ then, obtain probability density function of non-central Chi-square distribution using moment generating function.

OR

State and prove the important properties of the non-central Chi-square distribution.

(b) Define non-central 'F' distribution with (n_1, n_2) degrees of freedom. In usual notations obtain probability density function of non-central 'F' distribution.

OR

Define non-central 't' statistic. In usual notations obtain probability density function of non-central 't' distribution.

3. (a) Obtain the joint probability density function of the largest and the smallest order statistics.

OR

Define the sample range. Obtain the distribution of sample range for infinite range population. State the distribution of sample range for finite range population.

(b) If a random sample of size 'n' is taken from the exponential distribution with mean 1/3 then find the probability that the sample range does not exceed 2.

OR

Let a random variable 'X' follows an exponential distribution with mean θ, $\theta > 0$ random sample of size n is taken from this distribution then show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independently distributed.

4. (a) If $X(n) = \max\{X_1, X_2, \ldots X_n\}$ then show that

$$E(X_{(n)}) = E(X_{(n-1)}) + \int_0^\infty F^{(n-1)}(x)\,(1 - F(x))\,dx$$

OR

Define rank-order statistics with appropriate example. Give functional definition of rank-order statistics. In usual notations obtain the formula for the correlation coefficient between the rank-orders and variate values.

(b) Obtain the mean and variance of r^{th} order statistic for the uniform distribution U(0, 1).

OR

Explain the procedure of obtaining C.I. for p^{th} Quertile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a r.s. of size 7 taken from any continuous distribution with cdf $F_X(x)$ then obtain $P(X_{(3)} < \text{Population median} < X_{(5)})$

5. (a) Choose the correct answer :

(1) If $x_1, x_2, \ldots x_m, x_{m+1}, \ldots, x_{m+n}$ are independent normal variates with zero mean and standard deviation σ then the distribution of $\sum_{i=1}^{m} X_i^2 / \sum_{i=m+1}^{m+n} X_i^2$ is

(a) F(m, n) (b) F(m, m + n)

(c) F'_λ (m, n) (d) None of these

(2) If $x_1, x_2, \ldots x_n$ are independent variates each distributed as $N(0, \sigma^2)$ then the probability density function of $w = X_1 / \left(\frac{1}{n}\sum_{i=1}^{n} X_i^2\right)^{1/2}$ is

(a) 't' with n degrees of freedom

(b) 't' with (n – 1) degrees of freedom

(c) Non-central 't' with n degrees of freedom

(d) None of these

(3) If a random variable X has a chi-square distribution with degrees of freedom 'r' and a random variable Y has a non-central chi-square distribution with degrees of freedom 1 and non-centrality parameter λ then the distribution of the random variable $Z = X + Y$ is

(a) Chi-square with degrees of freedom r + 1

(b) Non-central chi-square distribution with degrees of freedom r + 1 and non-centrality parameter λ.

(c) Chi-square with degrees of freedom r

(d) None of these

(4) A non-central chi-square distribution is a

(a) Weighted sum of chi-square variables with weight as Poisson probabilities

(b) Weighted sum of Poisson variables with weight as chi-square probabilities

(c) Compound distribution of Poisson and chi-square distributions.

(d) (A) and (C) but not (B)

(5) The probability mass function of the Poisson Bionomial distribution is (A)

(a) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\ r}{x} p^x q^{n\,r-x} \frac{\lambda^r}{r!}$

(b) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\ r}{x} p^{-x} q^{n\,r-x} \frac{\lambda^r}{r!}$

(c) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\ r}{x} p^x q^{-n\,r-x} \frac{\lambda^r}{r!}$

(d) $P(x) = e^{-\lambda} \sum_{r=0}^{\infty} \binom{n\ r}{x} p^x q^{n\,r-x} \frac{\lambda^{-r}}{r!}$

(6) The probability generating function of the Poisson distribution is

(a) $G(Z) = e^{\lambda + \lambda e^{-m + mz}}$

(b) $G(Z) = e^{-\lambda} + \lambda e^{-m + mz}$

(c) $G(Z) = e^{\lambda} + \lambda e^{-m + mz}$

(d) $G(Z) = e^{-\lambda + \lambda e^{-m + mz}}$

(7) The probability generating function of the Poisson Negative Binomial distribution is

(a) $G(Z) = e^{-\lambda - \lambda (q - pz)^{-n}}$

(b) $G(Z) = e^{\lambda - \lambda (q - pz)^{-n}}$

(c) $G(Z) = e^{-\lambda + \lambda (q - pz)^{-n}}$

(d) $G(Z) = e^{-\lambda + \lambda (q + pz)^{-n}}$

(8) The recurrence relation for the probability for the Neyman type-A distribution is

(a) $P_{r+1} = \dfrac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \dfrac{m^j}{j!} P_{r-j}$

(b) $P_{r+1} = \dfrac{\mu_1' e^{-m}}{r-1} \sum_{j=0}^{r} \dfrac{m^j}{j!} P_{r-j}$

(c) $P_{r+1} = \dfrac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \dfrac{m^j}{j} P_{r-j}$

(d) $P_{r+1} = \dfrac{\mu_1' e^{-m}}{r+1} \sum_{j=0}^{r} \dfrac{m^j}{j!} P_{r-1}$

(9) Which one of the following statement is not true ?

(a) When 'v = 1', student's t distribution tends to Weibull distribution. (False)

(b) When 'v = 1', student's t distribution tends to Cauchy distribution. (true)

(c) The sampling distribution of F-statistic does not involve any population parameter. (true)

(d) The non-central chi-square distribution is the mixture of central Chi-square distribution and Poisson distribution. (True)

(10) Which one of the following statement is not true ?

(a) For Poisson Binomial distribution mean is less than variance. (True)

(b) For Poisson Pascal distribution mean is less than variance. (True)

(c) Neyman type-A distribution tends to Neyman type-B distribution. (False)

(d) Neyman type-B distribution tends to Neyman type-A distribution. (true)

(b) Answer the following questions. (In one sentences only)

(1) If a random sample of size '5' is taken from uniform distribution U(0,1) then write the probability density function of the sample median.

(2) If X ~ N(μ, 1) and Y is an independent chi-square variate with n degrees of freedom then write the distribution $t = X / \sqrt{Y / n}$.

(3) Write applications of Contagious Distribution.

(4) Define ascending factorial moment generating function.